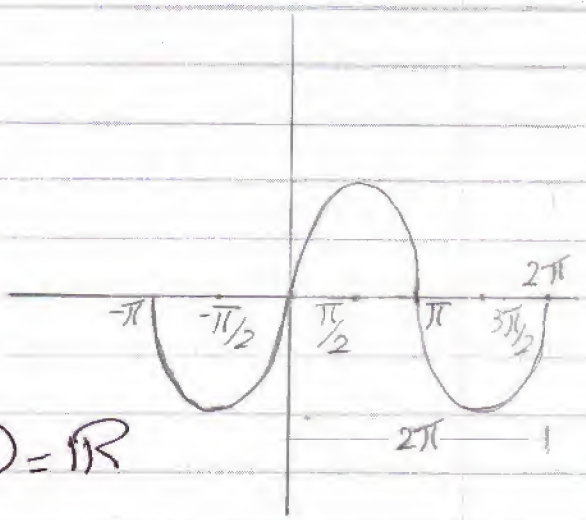
Lec. 4

تفاضل د. عزة

الدوال المثلثية:

1] $y = \sin x$



$D = \mathbb{R}$

$R = [-1, 1]$

$-1 \leq \sin x \leq 1$

odd $\rightarrow \sin(-x) = -\sin x$

2π = الدورة

$\therefore \sin(x + 2n\pi) = \sin x$

$n = 0, \pm 1, \pm 2, \ldots$

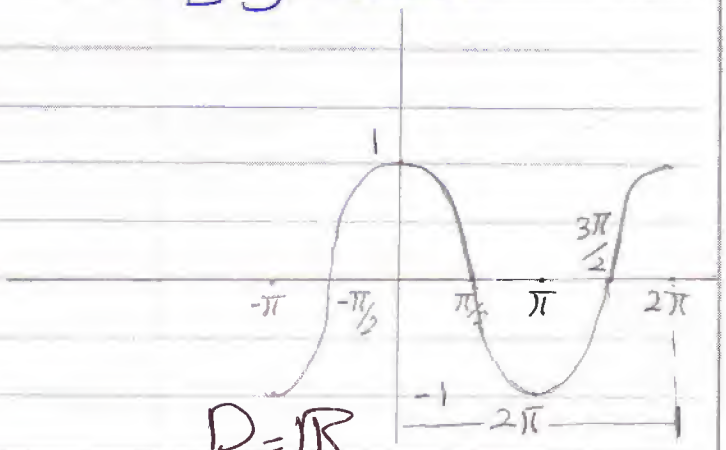
$\in \mathbb{Z}$ الأعداد الصحيحة

$\sin x = 0$, when $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

$\therefore x = n\pi$

$n \in \mathbb{Z}$

2] $y = \cos x$



$D = \mathbb{R}$

$R = [-1, 1]$

$-1 \leq \cos x \leq 1$

even $\rightarrow \cos(-x) = \cos x$

$\cos(x + 2n\pi)$

$n \in \mathbb{Z}$

$\cos x = 0$

$x = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2, \ldots$

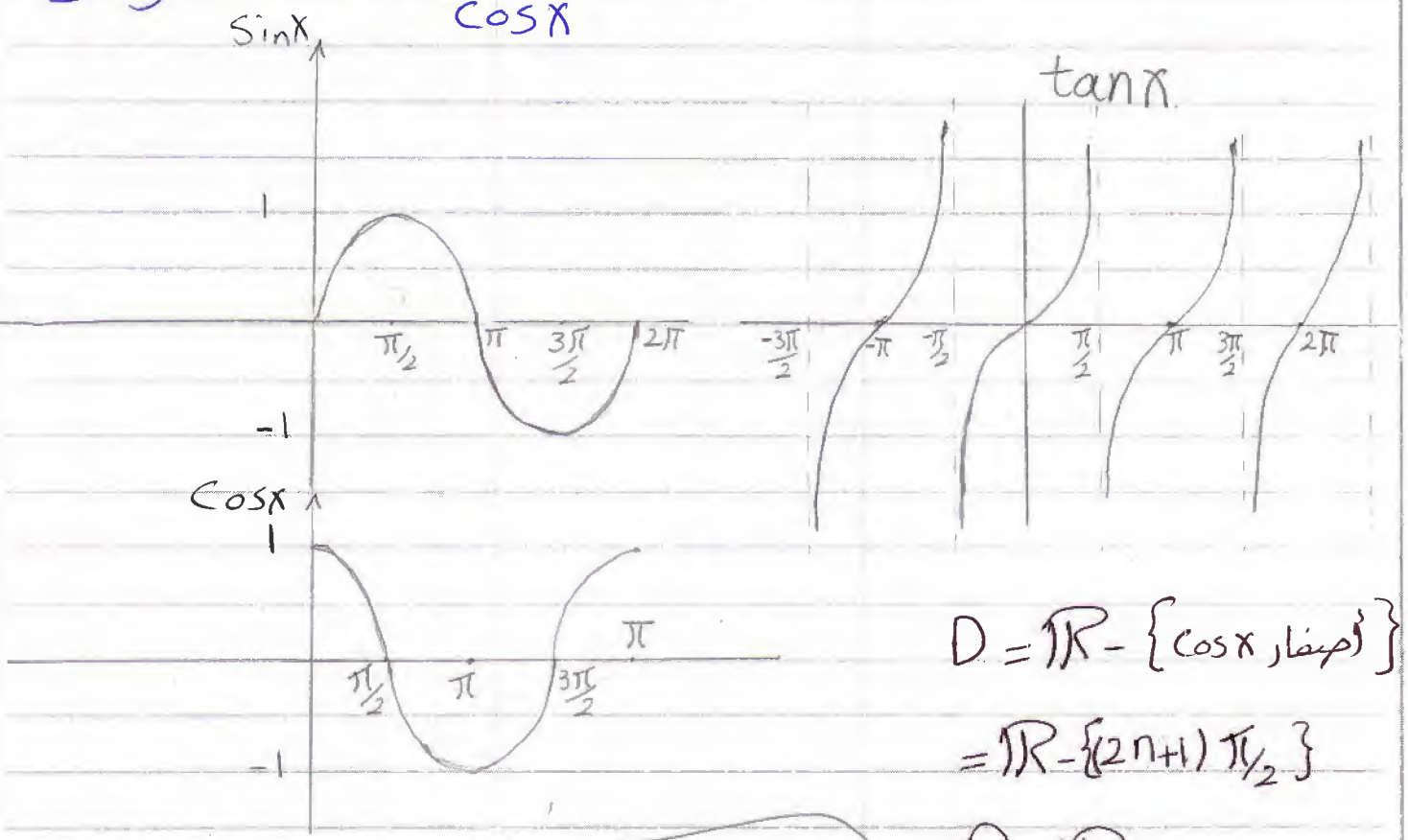
$= (2n+1)\pi/2$

$n \in \mathbb{Z}$

$$\cos(x - \pi/2) = \sin x$$

$$\sin(x + \pi/2) = \cos x$$

3] $y = \tan x = \frac{\sin x}{\cos x}$



$D = \mathbb{R} - \{\text{أصفار } \cos x\}$

$= \mathbb{R} - \{(2n+1)\pi/2\}$

$R = \mathbb{R}$

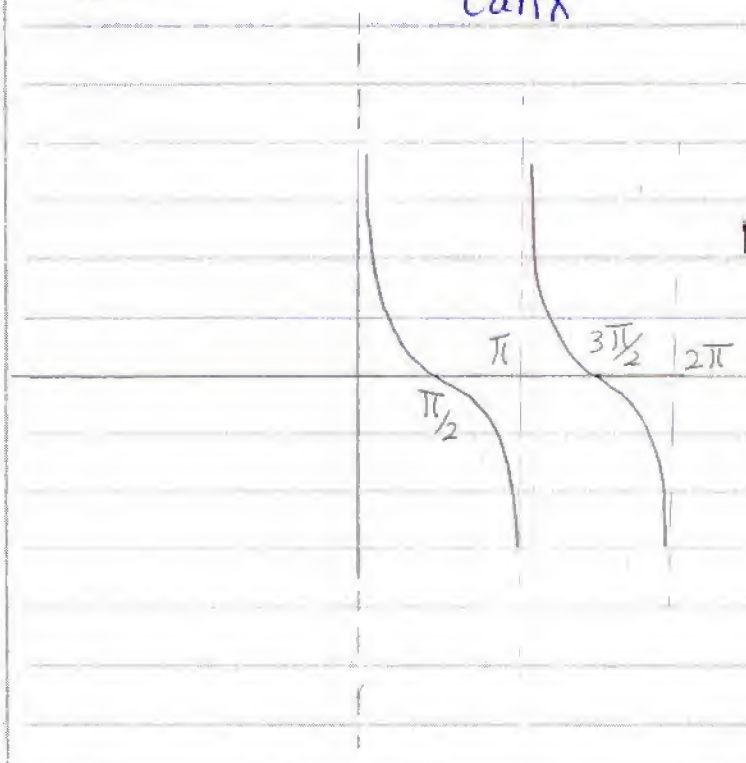
odd = $\frac{\text{odd}}{\text{even}}$

الدورة = π

$\tan(x + n\pi) = \tan x$

$n \in Z$

4] $y = \cot x = \frac{1}{\tan x}$



$D = \mathbb{R} - \{\text{أصفار } \sin x\}$

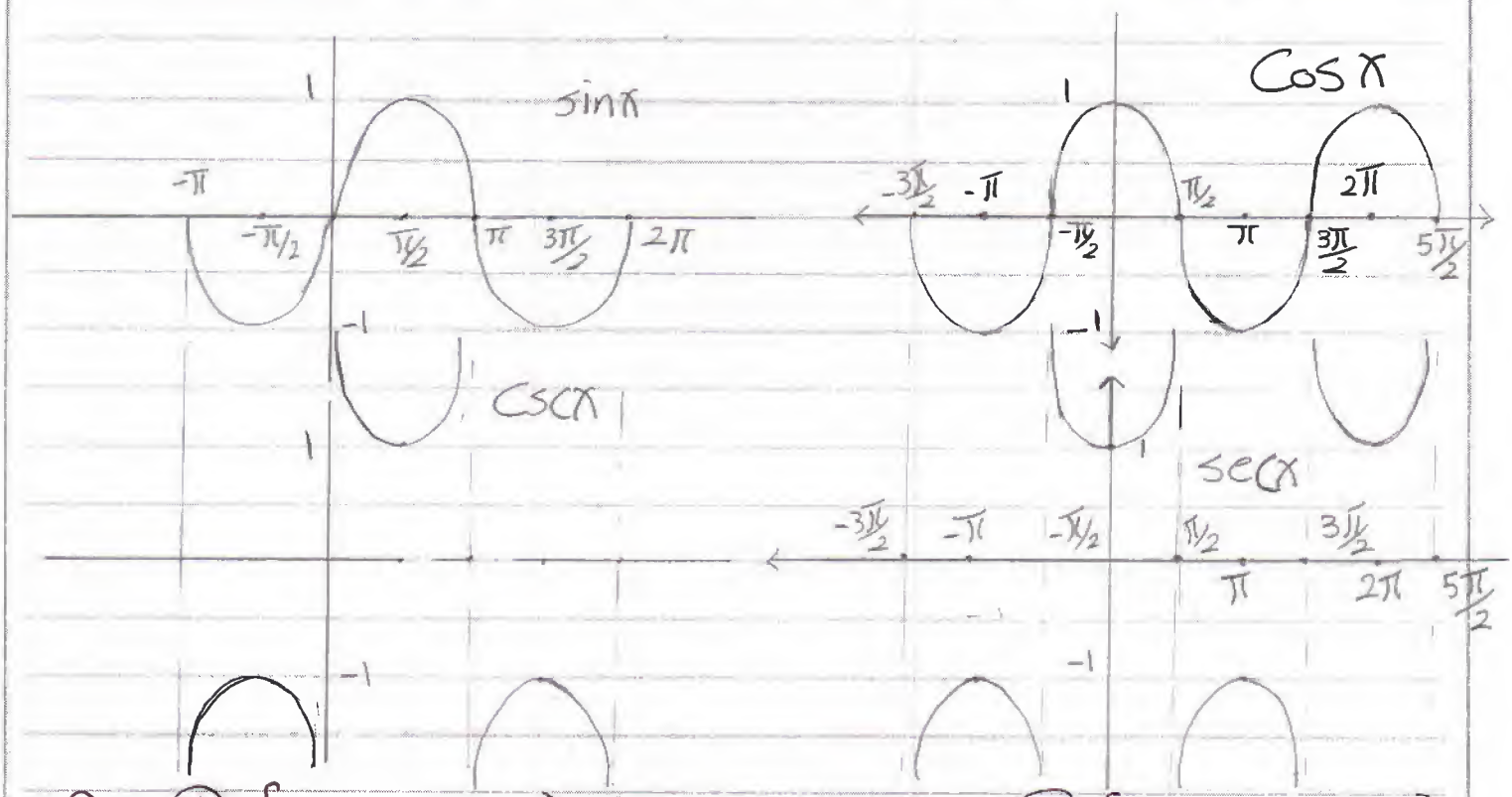
$= \mathbb{R} - \{n\pi\}$

$R = \mathbb{R}$

odd

5] $\csc x = \frac{1}{\sin x}$

6] $\sec x = \frac{1}{\cos x}$



$D = \mathbb{R} - \{\sin x \text{ اصفار}\}$

$\mathbb{R} - \{n\pi\}$

$R = \mathbb{R} - (-1, 1)$

odd

2π الدورة

$D = \mathbb{R} - \{\cos x \text{ أصفار}\}$

$= \mathbb{R} - \{(2n+1)\pi/2\}$

$R = \mathbb{R} - (-1, 1)$

even

2π الدورة

الدالة الأسية العامة: Generalized exponential fn.

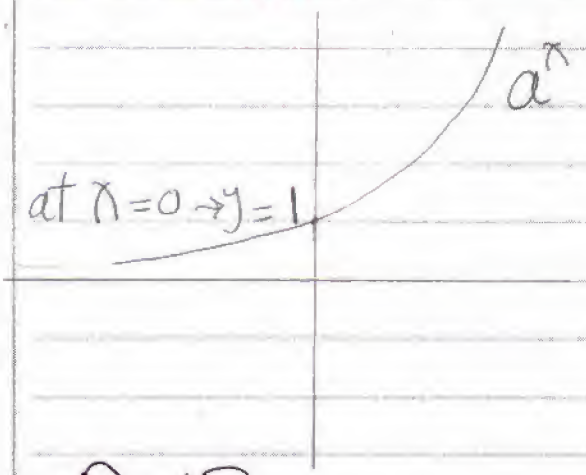
$y = a^x$ → المتغير هو الأس

a → عدد ثابت

$a > 0$

$y = x^n$ → ثابت

x → الأساس هو المتغير



تذكر

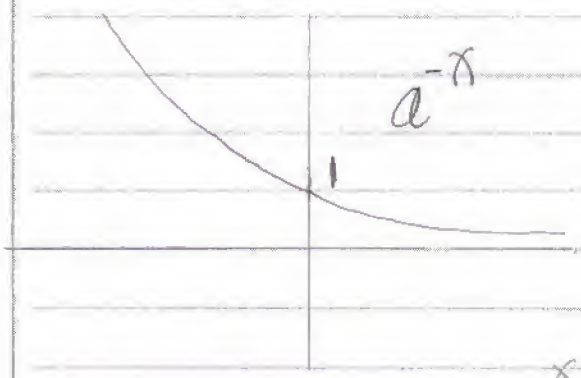
$$\boxed{\begin{aligned} a^x \cdot a^y &= a^{x+y} \\ \frac{a^x}{a^y} &= a^{x-y} \\ (a^x)^y &= a^{xy} \end{aligned}}$$

$D = \mathbb{R}$

$R = (0, \infty)$

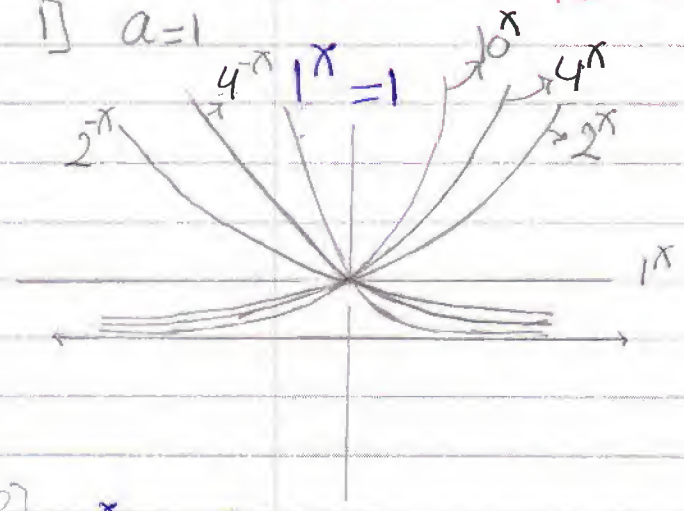
$a^{\infty} = \infty$

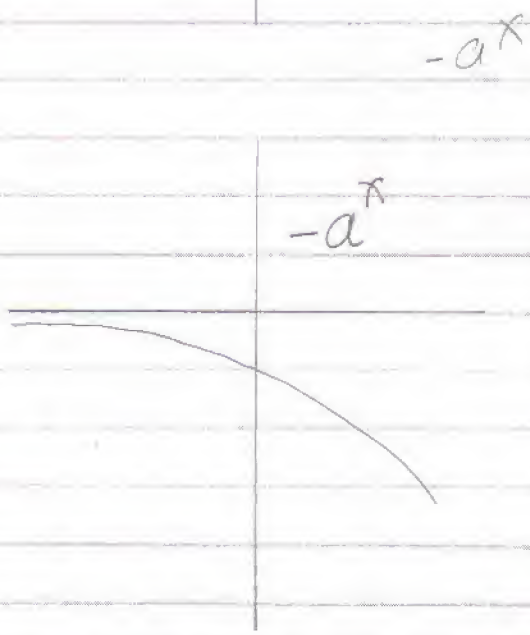
$a^{-\infty} = 0$



المنحنى عند قيم a المختلفة:

1] $a = 1$



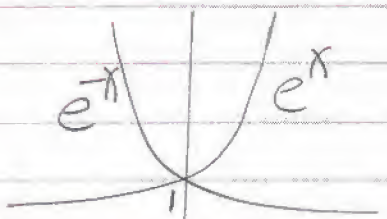


2) $\left(\frac{1}{2}\right)^x = 2^{-x}$

$\left(\frac{1}{4}\right)^x = 4^{-x}$

3] $a = e \simeq 2.71782$ Euler ثابت أويلر

$y = e^x$ Natural الدالة الأسية الطبيعية



$x \simeq 13.8$

$e^x = e^{13.8} > 10^8$

$e^0 = 1$

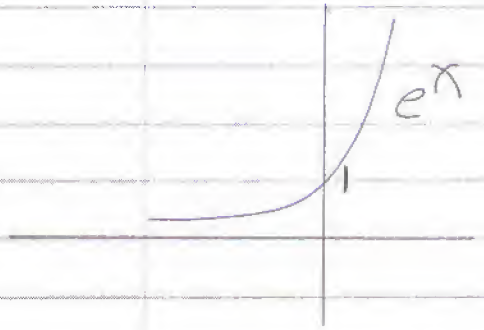
$e^\infty = \infty$

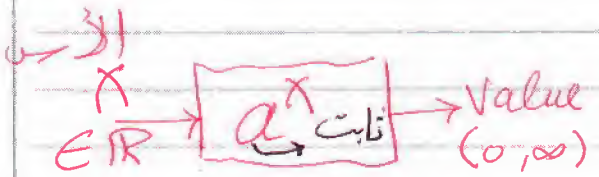
$e^{-\infty} = 0$

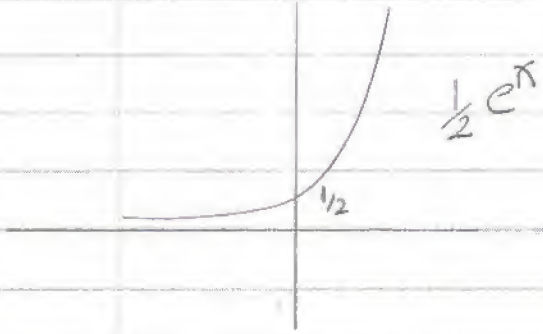
$y = \frac{1}{2} e^x - 1$ أزح منحنى 1

$D = \mathbb{R}$

$R = (0, \infty)$



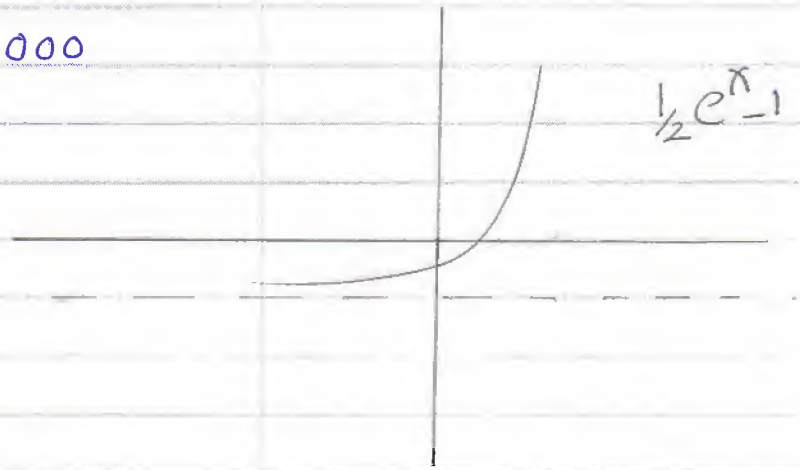




$\log_{10} 1000 = 3 \longleftrightarrow 10^3 = 1000$

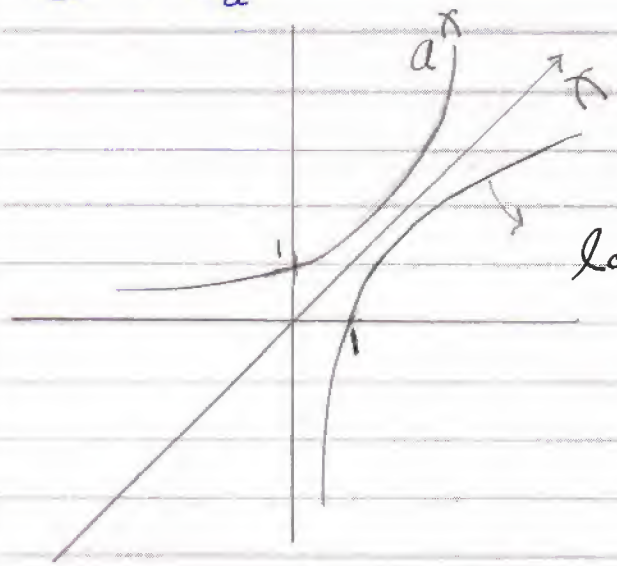
$\log_{10} 0.001 = -3$

$\log_2 16 = 4$



الدالة اللوغاريتمية العامة:

$y = \log_a x$



$D = (0, \infty)$

$R = \mathbb{R}$

$\log_a 1 = 0$, because $a^0 = 1$

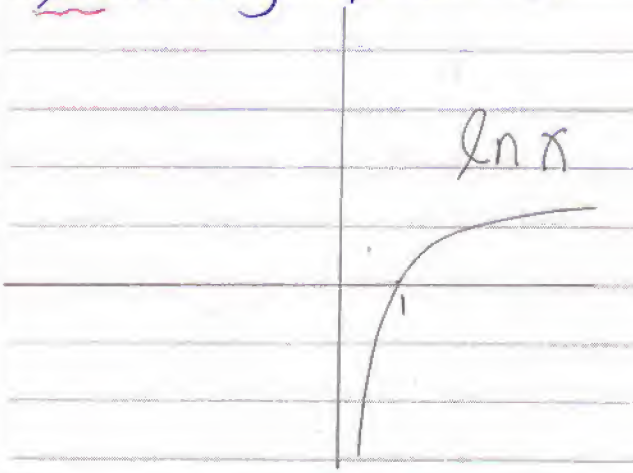
$\log_a \infty = \infty$

$\log_a 0 = -\infty$

الدالة اللوغاريتمية الطبيعية:

$y = \log_e x \xrightarrow{\text{inverse}} y = e^x$

$\ln x = y \rightarrow e^y = x$



$D = (0, \infty)$

$R = \mathbb{R}$

$\ln 1 = 0$

$\ln \infty = \infty$

$\ln 0 = -\infty$

$f^{-1}(f(x)) = x$

$\ln e^x = x = x \ln_e e = x$

$f(f^{-1}(x)) = x$

تذكر

$\ln(xy) = \ln x + \ln y$

$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

$\ln x^n = n \ln x$

$y = \ln(x-2) - 1$ → مع نفسك

$$\boxed{e^{\ln x} = x}$$